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ABSTRACT

A project is described that provides explicit instruction on the skills necessary for solving word problems. An informal inventory of such skills is being developed, intended for diagnostic use. It contains six sections, each keyed to the problem-solving skills of understanding the problem, representing the unknown, writing the equation, and solving the equation. In Fall 1985, a draft of the inventory was given to 130 students at South Dakota State University enrolled in two mathematics courses. Results indicated the skills on which students needed work. In addition, a pre/post test on problem solving was designed for use in future experiments. It was administered to 105 students; internal consistency estimates ranged from .63 to .65 for the two forms. Comments are included on the test as a whole and on particular items. Revision will continue. The inventory and tests are appended. (MNS)

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Skills for Solving Word Problems: Testing and Teaching Them

A progress report of a project being conducted at South Dakota State University

by

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Report delivered at
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Jackson, Wyoming
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SE 046 408

Skills for Solving Word Problems: Testing and Teaching Them

Word problems have been and continue to be the bane of math education. In spite of the apparent value of acquiring problem-solving skills, it's still typical for high school and college students to shun word problems, to prefer instead exercises where answers are obtained by the application of a limited routine of principles.

Certainly part of the fault for this situation lies in the instruction which students have received as they progressed through math classes. Word problems are often ignored or treated as "supplementary" activities; and, when they are taught, the instruction often consists of little more than introductions and practice exercises. Thus, rather than receive explicit instruction in problem-solving skills, students receive problems to be solved on the apparent assumption that doing the problems will result in skill growth. Some students, of course, do develop skills through this method. But far too many develop little other than an attitude that solving word problems is too obtuse and irrelevant to be worth the effort.

We believe that part of the remedy for this situation must include explicit instruction of the skills necessary for solving word problems; and we further believe that explicit skill instruction implies, first of all, that teachers know and

understand these skills and be able to diagnose students' skill needs. Toward the goals of better understanding, diagnosis, and eventually better instruction, we have joined efforts in a project that will begin with teachers and students at the college level and extend, hopefully, into junior and senior high.

Project Progress Report

As a beginning, we are developing an "Informal Inventory of Skills for Solving Word Problems." It's based on the informal reading inventory used by reading educators in the sense that it (a) is informal, i.e., non-standardized; (b) is based on about four or five skills or strategies considered necessary in the completion of a task--in reading, the task of comprehension, in our case, the task of solving word problems; (c) contains only questions keyed to specific skills and following the lead of informal reading inventories usually consists of five questions to test each skill; (d) is intended to be used for diagnostic purposes, i.e., the determination of individual as well as group strengths and weaknesses in relation to specific skills; (e) is intended to yield results which will lead to instruction that better meets the needs of individuals and groups.

A sample copy of a "Informal Inventory for Solving Word Problems" is attached (Appendix A). This particular one was specifically designed for and used with students at South Dakota State University fulfilling their basic math requirements (Math 111)

or those taking the developmental course (Math 019) that preceeds the Math 111. The inventory contains six sections, each section being keyed to the following word problem solving skills.

SECTION A — Understanding the problem, its parts and what must be found.

SECTION B, C, D — Representing the unknowns by a letter. (In Section B, the focus was on determining what variable should be represented by X. In Section C, the variable to be represented by X was given, and respondents were asked to represent the other variables. In D, the representations all involved visualizing or diagramming.)

SECTION E — Writing the equation.

SECTION F — Solving the equation.

These skills were chosen simply because they correspond with the schema for solving word problems presented in the Math 111 textbook being used in the classes where we field-tested the instrument. It must be emphasized that other skill taxonomies could constitute a basis for an inventory like this, as long as one were able to write questions which would discretely test a given skill. Likewise a similar inventory could be written for different age and experience levels.

In the Fall of 1985, this inventory was given to 130 students at South Dakota State University. Ninety-one (Group A) were

enrolled in Math 111. Thirty-nine (Group B) were enrolled in Math 019. The mean percent correct were as follows.

	A	B
Skill 1 (SECTION A)	92	92
Skill 2 (SECTIONS B, C, D)	64	50
Skill 3 (SECTION E)	43	22
Skill 4 (SECTION F)	64	36

The inventory was given immediately before students began working with the unit on solving word problems. Some of the diagnostic information provided can be seen from the chart above. For example, these students (unlike high school students who took a similar test in Spring, 1985) probably don't need any extra instruction on the understanding of word problems and what's being asked for, but they certainly need help in writing equations (Skill 3), especially those in Group B.

In addition to this kind of group information, information can be obtained for individual students. Consider, for example, the following mean percent for selected students (as indicated by numbers across the top).

	Student 41	Student 95
Skill 1	92	96
Skill 2	31	71
Skill 3	80	40
Skill 4	70	10

Based on this information (and it is always recognized that an inventory is only one source of information), these two students have different needs. Student #41 appears to need special help with representing variables. In contrast, student #95 had considerably more troubles with the writing and solving of equations.

It must be emphasized that the attached inventory is to be considered a draft copy. It was given this fall as a pilot for more serious work in the spring. At the present the results have been only informally analyzed, so it's too early to make announcements about revisions. However, even a casual item analysis has led us to question--

(a) the value of separating Skill 2, "representing variables," into three sections. If the test can be made shorter, it should be; and if a factor analysis doesn't justify the three subsections, they will be blended into one and fewer total questions will be needed.

(b) the worth of question 4, Section C and question 3, Section D. An inordinate percentage of people missed these questions, or parts of them. Why? And has it skewed the results?

(c) whether our attempts to include representatives of most types of word problems (age, motion, etc.) will yield the information it was intended to yield about strengths

and weaknesses in terms of problem types; or whether some problem types, such as consumer applications, require such specific strategies that testing them confounds gaining an accurate picture of students' abilities to employ general word problem solving skills.

In addition to the inventory that was field-tested this fall, a pre/post test was designed for use with future experiments at the college level (Appendix B). The sample consisted of 105 students enrolled in Math 111 Algebra at South Dakota State University. Two sections were chosen because neither instructor had begun the chapter on word problems. The students in both sections were randomly given one of the two forms of the test.

Form A was administered to 49 students and Form B to 56 students. The basic types of problems were numbers which include consecutive odd/even numbers, plane geometry, finance, as well as rate, distance and time. Two forms of the instrument were developed each containing seven problems. Two parallel forms were constructed for use as pre/post test instruments. The administration took 30 minutes. The internal consistency estimate using the Kuder-Richardson Formula 20 was .66 for Form A and .63 for Form B when the instruments were scored looking only for the correct solution. The instruments were also scored with a correct solution requiring the appropriate algebraic equation. In this case the internal consistency estimate was .64 for Form A and .63 for Form B.

Item analyses were undertaken to identify faulty items. In the analysis of the two forms where answers only were scored there appear to be four items that might not be parallel items.

Insert Table 1 about here

Item 1 on Form A requires the student to find two numbers as opposed to finding a single number in Item 1 on Form B. An examination of student work indicates that this was the case for two students only.

Items 3 resulted in the largest discrepancy of 17 percentage points. The immediate cause of this discrepancy would appear to be the difference in the size of the perimeters. The problem on Form B would be much easier to solve by trial and error. This is supported by the item analysis where correct equations and solution were required.

Items 4 appear to be quite similar with respect to size of numbers used and variable representation. Form A, Item 4 does require the use of the distributive property. An examination of student work does not indicate an apparent reason for the discrepancy. Six students taking Form A used 360° in their solution compared to one student taking Form B. These items were often left blank.

Items 7 differed by 14 percentage points. The item on Form B

requires division with the insertion of a zero ($12\overline{)1260}$), but only three students made this mistake. Eighteen students, however, multiplied .12 times 1260 rather than dividing. This was consistent with responses on Form A where 15 students chose to multiply rather than divide.

In the analysis of the two forms where the algebraic equation was required with the correct solution items 1, 4 and 7 are consistent with the above form of scoring.

Items 2 differ by 10 percentage points. An examination of student work indicates that it was easier to use trial and error to answer the Form B item without the use of an equation.

Insert Table 2 about here

Conclusion

At this point, we have field-tested two instruments with college students. The instruments will be revised as necessary and used in the Spring, 1986 semester as part of our project. From four to six basic math classes will be involved this spring. All will take the pre-test and--after the given instructional period--all will take the post test. Only those in the experimental groups will be given the inventory. Inventory results will be shared with the teachers of these experimental groups as well as specific strategies for providing explicit skill instruction in areas of

need. The control groups will receive the instruction which is more traditional at this university. Pre/post test gain will be used as indicators of the effects of the treatment.

Table 1

Reliability Coefficients and Item Analysis for the Combined Group Requiring Correct Answer Only

	n	KR 20 Coefficient
Form A	49	.658
Form B	56	.633

Percentage Correct Response

Item	Form A	Form B
1	80	91
2	82	79
3	31	48
4	43	34
5	12	16
6	43	41
7	53	39

Table 2

Reliability Coefficients and Item Analysis for the Combined Group Requiring Algebraic Equation

	n	KR 20 Coefficient
Form A	49	.639
Form B	56	.628

Percentage Correct Response

Item	Form A	Form B
1	55	64
2	35	25
3	27	25
4	37	27
5	0	2
6	4	4
7	29	14

A

NAME _____

INFORMAL INVENTORY OF SKILLS FOR SOLVING WORD PROBLEMS

This inventory is intended to measure skills used for solving word problems. It is NOT a test, and it will NOT be graded. It is an attempt to identify areas in which more instructional help may be needed, and the results will be used to help design this instruction.

You will find on the next pages a series of problems. Following each problem will be a question. Read the problem, then respond only to the question about the problem. Don't solve the whole problem unless that's what is called for. For example:

PROBLEM:	The length of a rectangle is 5 cm more than the width. The perimeter is 50 cm. Find the length and width.
QUESTION:	Write an equation for this problem letting w be the width and $w + 5$ be the length.
RESPONSE:	$2w + 2(w + 5) = 50$

Notice in the above sample that the proper response was to write only the equation. According to the question that's all that was necessary.

Please complete the inventory carefully, but, if possible, in the time allotted. The results will be helpful for future instruction.

SECTION A

PROBLEM: The perimeter of a rectangle is 52 m. If the length is 4 m more than the width, find the dimensions.

QUESTION: Based on the above problem, determine whether the following statements are true or false. (Circle the correct response in the parentheses.)

a. The distance around the rectangle is unknown. (T/F)

b. The length and width are not equal. (T/F)

c. If the problem would be solved correctly, then a person would know the length and width of the rectangle. (T/F)

PROBLEM: Two cars leave one point going in opposite directions. The second travels 10 mph faster than the first; after 3 hours they are 300 miles apart. How fast is each car going?

QUESTION: Based on the above problem, answer the following: (Circle the correct responses.)

a. The cars both started at the same time. (T/F)

b. The cars both started from the same place. (T/F)

c. The cars both traveled the same distance. (T/F)

d. Which of the following is the question asking for:

a. distance

b. rate

c. time

PROBLEM: The sales tax rate in Pukwana, SD is 4%. How much tax would be charged on a purchase of $42? What is the total cost?

QUESTION: The above problem contains two questions. Which one of the following statements would be true about the two questions? (Circle the letter of the correct response.)

a. The answer to the first question would involve a larger amount of money than the answer to the second.

b. The answer to the second question would involve a larger amount of money than the answer to the first.

c. The answer to the first question would be the same as the answer to the second question.

PROBLEM: The sum of two numbers is 52. The larger is three times the smaller number. Find the two numbers.

QUESTION: Restate in your own words what you are supposed to find in this problem. Then describe the information you have been given to help you find it.

PROBLEM: The sum of Karen's age and Sam's age is 78 years. If Karen is 6 years younger than Sam, how old is each?

QUESTION: Circle the correct response in the parenthese.

a. Karen is older than Sam. (T/F)

b. Karen's age added to Sam's age would result in 78 years. (T/F)

c. To answer this problem, a person would have to figure out the age of Karen and the age of Sam. (T/F)

SECTION B

PROBLEM: The sum of two consecutive integers is 47. Find the integers.

QUESTION: How would you represent the unknown numbers? (Circle the letter of the correct response.)

a. $x, x + 2$

b. $x, x + 1$

c. $x, x - 2$

PROBLEM: A sport coat is discounted 30% and the sale price is $87.50. What was the original price?

QUESTION: Which of the following should be represented by x? (Circle the letter of the correct response.)

a. The amount of discount.

b. The original price.

c. The sale price.

PROBLEM: Tom is four times as old as Jim, and the difference of their ages is 33 years. How old is each?

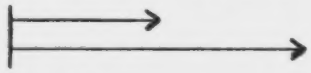
QUESTION: How would you represent the ages? (Circle the letter of the correct response.)

	JIM	TOM
a.	$4x$	x
b.	x	$4x$
c.	x	$x + 4$
d.	x	$x - 4$

SECTION D

PROBLEM: Two cars leave the same point, one traveling east and the other traveling west. If the first is moving 10 mph faster than the second, and if after 4 hours they are 424 miles apart, then how fast is each traveling?

QUESTION: Which of the following would be the best visual representation of the problem? (Circle the letter of the correct response.)

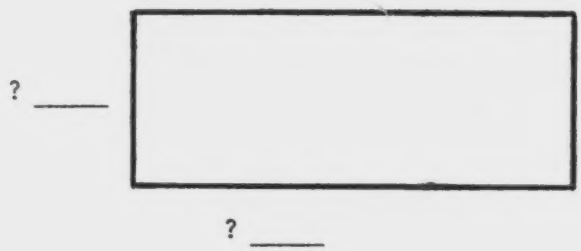
a. 

b.

c.

PROBLEM: The perimeter of a rectangle is 94 feet. If the length is 8 feet more than twice the width, find the dimensions.

QUESTION: Below is a visual representation of the problem. Label the picture below in terms of one variable.



PROBLEM: Becky can ride her bike to school in 10 minutes, but it takes her an hour if she walks. Her speed walking is 15 mph slower than her speed when riding. Find each rate.

QUESTION: Below is a chart which might be used to help solve the problem. Fill in the chart with information from the problem above.

	d = r . t		
Riding			
Walking			

SECTION C

PROBLEM: The second angle of a triangle is six times as large as the first angle. If the third angle is 45° more than twice the first, find the measure of each angle.

QUESTION: If we let x = the measure of the first angle, then how would the measures of the following be represented?

a. The measure of the second angle = _____.

b. The measure of the third angle = _____.

PROBLEM: A machine which was manufactured 15 years ago can produce 20 items per day. A more efficient machine, manufactured last year, can produce 45 items per day. How long will it take to produce 1105 items if both machines are turned on together?

QUESTION: If we let x = the number of days needed to produce 1105 items when machines work together, then how would the following numbers be represented?

a. Number of items produced by older machine in x days = _____.

b. Number of items produced by newer machine in x days = _____.

PROBLEM: A wire is 40 inches long. It is to be cut into three pieces in such a way that the second piece is three times as long as the first piece and the third piece is 5 inches longer than the first piece. Find the length of each piece.

QUESTION: If x = the length of the first piece, then how would the length of the other pieces be represented?

a. The second piece _____.

b. The third piece _____.

PROBLEM: When his uncle died, Martin received an inheritance of $25,000.00. He put part of the money in an account which paid 14% simple interest, and the rest into a stock which paid a 17% annual dividend. If at the end of the year he had a total income of $3920.00 from the two, how much was invested in each category?

QUESTION: If x = the amount of money invested at 14% then how would the 17% investment be represented?

SECTION E

PROBLEM: A basketball player made 9 free throws in 15 attempts. What was her free throw percentage?

QUESTION: If we let x = the free throw percent, then which of the following would be the proper equation? (Circle the letter of the correct response.)

a. $x = \frac{9}{15}$

c. $x = \frac{9}{15} \div 100$

b. $100x = \frac{9}{15}$

d. $\frac{x}{100} = \frac{9}{15}$

PROBLEM: If three times a number is subtracted from 20, the result is twice the number.

QUESTION: Assume that x = the desired number and 20 - 3x = three times the number subtracted from 20.

Then which of the following would be the proper equation for the problem? (Circle the letter of the correct response.)

a. $x\ (20-3x) = 20$

c. $20 - 3x = 2x$

b. $20x = 3\ (20)$

d. $x = 20 - 3x$

PROBLEM: If the second angle of a triangle is 50° more than the first angle, and the third angle is eleven times the first, determine the measure of each angle.

QUESTION: Assume that for the above problem, the variables were represented as follows:

x = first angle

$x + 50$ = second angle

$11x$ = third angle

Given these, write the equation needed to solve this problem in the space below.

PROBLEM: Two truckers leave San Francisco at the same time heading east. The first is traveling at a rate of 60 mph and the second at 54 mph. How far apart will they be in 7 hours?

QUESTION: Which of the below would be the proper equation for the problem?

a. $60(7) - x = 54(7)$

b. $60(7) + 54(7) = x$

c. $7(60 - 54) = x$

d. $60x - 54x = 7$

PROBLEM: Tom is twice as old as Sue, and Sue is three years older than Jim. The total of their ages is 85 years. How old is Jim?

QUESTION: Assume that x = Sue's age
$2x$ = Tom's age
$x-3$ = Jim's age

Then write the proper equation below.

SECTION F

Solve each of the following equations. Note that some offer a multiple-choice response. In those problems you need only circle the correct answer.

1. $20 - 3x = 2x$

   a. 6

   b. 4

   c. 18

2. $7 (2y - 1) = 3 + 14 y$

3. $x + 6x + (2x + 45) = 180$

   a. 15

   b. 28

   c. 67

4. $20 x + 45 x = 1105$

5. $x - (0.30) x = 87.50$

   a. 602

   b. 125

   c. 78

6. $.06x = .08\ (20000 - x)$

7. $3r + 3(r + 10) = 300$

   a. 45

   b. 50

   c. 55

8. $5x + 6 = 7x - 14$

   a. $\frac{5}{3}$

   b. 8

   c. 10

9. $.06x + x = 11130$

10. $.12x = 1140$

    a. 95

    b. 9500

    c. 950

B

Form A Show all work! Name _____

1. There are two numbers whose sum is 72. One number is twice the other. What are the numbers?

2. Four consecutive odd integers have a sum of 64. Find the integers.

3. A rectangle has a length which is 4 feet less than three times the width. The perimeter is 224 feet. What are the dimensions?

4. The second angle of a triangle is 20 degrees greater than the first angle. The third is twice the second. Find the three angles.

5. Mrs. Silver inherited $20,000 which she invested in stocks and bonds. The stocks paid 8% and the bonds 6%. If the return in the stocks was $80 less than that received from the bonds, how much did Mrs. Silver invest in each?

6. Two cars are headed for Las Vegas. One is 50 miles ahead of the other on the same road. The one in front is traveling 60 mph while the second car travels 70 mph. How long before the second car overtakes the first car?

7. Mary spent $4400 for a used car. This was 22% of her annual income. What was her annual income?

Form B Show all work! Name _____

1. There is a number such that three times the number minus 6 is equal to 45. Find the number.

2. The sum of four consecutive even integers is 44. What are the numbers?

3. The length of a rectangle is 5 feet more than twice the width. The perimeter is 28 feet. Find the dimensions.

4. The first angle of a triangle is twice the second and the third is 5 degrees larger than the first. Find the three angles.

5. Mr. Gold invested $50,000, part at 6% and part at 8%. The annual interest on the 6% investment was $480 more than that from the 8% investment. How much was invested at each rate?

6. A freight train starts from Los Angeles and heads for Chicago at 40 mph. Two hours later a passenger train leaves the same station for Chicago traveling 60 mph. How long before the passenger train overtakes the freight train?

7. What are the total sales on which John received a commission of $1260 if his commission rate is 12%?